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A Detailed Prospect Theory Explanation of the Disposition Effect

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The disposition effect has been characterized in various ways: the "effect, whereby investors are anxious to sell their winners, but reluctant to sell their losers" (Shefrin [2005], pp. 419); "the tendency to hold losers too long and sell winners too soon" (Odean [1998], pp. 1775) and the effect, whereby investors "sell winners more readily than losers" (Odean [1998], pp. 1779). The most discernable aspect of these characterizations is their imprecision, particularly with regard to time. In what follows, we provide a detailed explanation of the disposition effect based on a straightforward application of prospect theory (Kahneman and Tversky [1992]; Kahneman and Tversky [1979, 2000]). The analysis begins with the traditional account of the disposition effect (Shefrin and Statman [1985]) and provides precise time-independent concepts that replace "sell too soon" and "hold too long." The analysis shows when the prospect theory explanation of the disposition effect requires only the valuation function and when the explanation requires both the valuation function and the probability weighting function.

Keywords: Disposition effect, S-Shape valuation function, Probability weighting function

INTRODUCTION—CHARACTERIZATIONS OF THE DISPOSITION EFFECT

Investors regularly make decisions whether to buy, sell or hold securities. It is well known that investors do not always act as expected utility maximizers. The disposition effect is one such divergence from expected utility theory. The disposition effect has been characterized in various ways: the "effect, whereby investors are anxious to sell their winners, but reluctant to sell their losers" (Shefrin [2005], 419); "the tendency to hold losers too long and sell winners too soon" (Odean [1998], 1775) and the effect, whereby investors "sell winners more readily than losers" (Odean [1998], 1779).

The disposition effect has been observed in traditional financial markets (Greenblatt and Keloharju [2001]; Odean [1998]; Shapira and Venezia [2001]), in real estate markets (Case and Shiller [1989]; Genovese and Meyer [2001]) and in experimental markets (Weber and Camerer [1998]). The disposition effect has been examined from various perspectives, including the individual investor (Weber and Camerer [1998]), groups of investors (Dhar and Zhu [forthcoming]; Feng and Seasholes [2005]; Locke and Mann [2005]; Shapira and Venezia [2001]) and the market (Oehler et al. [2003]; Shefrin [2005]).

The most discernable aspect of the characterizations of the disposition effect is their imprecision, particularly with regard to time. In what follows, we provide a detailed explanation of the disposition effect based on a straightforward application of prospect theory (Kahneman and Tversky [1979]; Tversky and Kahneman [1992]). The analysis begins from the traditional account of the disposition effect (Shefrin and Statman [1985]) and provides precise time-independent concepts that replace "sell too soon" and "hold too long."

Our analysis is limited to the roles played by investors' preferences, as represented the S-Shape valuation function, and investors' partial beliefs, as represented by the probability weighting function. Our analysis does not include other potential explanations of the disposition effect, such as fear, hope, regret, and pride (Fogel and Berry [2006]). The paper ends with a brief set of concluding remarks and some suggestions for further research.

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PROSPECT THEORY EXPLANATION OF THE DISPOSITION EFFECT

The logic behind the prospect theory explanation of the disposition effect is straightforward and is based on the concavity of the valuation function over gains and the convexity of the valuation function over losses. The standard argument is presented in Shefrin and Statman [1985] and Goldberg and von Nitzsch [2001]. Suppose an individual purchases a share of stock and enjoys a gain of x_0 or suffers a loss of $-x_0$ in the share price. The standard account presumes the individual considers small, equally likely, incremental changes in the share price, in amounts h and $-h$, respectively. ('Small' is modeled as $h < x_0$.) The standard account also presumes that the investor purchases exactly one stock. (See Shefrin [2005], p. 426, for a discussion of the multi-stock case.)

The standard account can be formulated as follows. Let v_+ and v_- be the components of the Kahneman-Tversky valuation function for gains and losses, respectively.[1] The Kahneman-Tversky valuation function is concave for gains, convex for losses, and steeper for losses than for gains. Since v_+ is concave,

$$v_+(x_0) - v(x_0 - h) > v_+(x_0 + h) - v_+(x_0),$$

and an investor who experiences a gain, x_0, anticipates a relative loss and therefore will sell instead of hold; and since v_- is convex,

$$v_-(-x_0 + h) - v_-(-x_0) > v_-(-x_0) - v_-(-x_0 - h),$$

and an investor who experiences a loss, $-x_0$, anticipates a relative gain and therefore will hold instead of sell. Our analysis considers only the combination of x_0 and h wherein h is smaller than x_0. In the Conclusion section we comment on the combination wherein h is greater than x_0. Figure 1 presents the combinations of x_0 and h considered here.

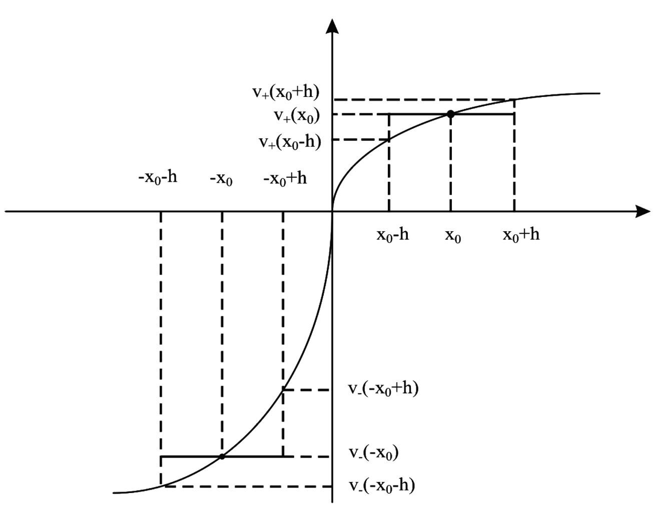

FIGURE 1 Disposition effect for small incremental changes in price.

TABLE 1
Payoffs after a gain x_0

	Sell	Hold	Probability
Incremental gain (h)	x_0	$x_0 + h$	p_{GAIN}
Incremental loss ($-h$)	x_0	$x_0 - h$	$1 - p_{GAIN}$

The foregoing graph represents two distinct decision problems. The decision problem after a gain x_0 is given in Table 1, and the decision problem after a loss $-x_0$ is given in Table 2.

Note that $p_{GAIN} = \text{Prob}(h/x_0)$ and $p_{LOSS} = \text{Prob}(h/-x_0)$. Also note that p_{GAIN} and p_{LOSS}, and the related decision problems, pertain to distinct worlds, the former where the individual realizes a gain, in the amount x_0, and the latter where the individual realizes a loss, in the amount $-x_0$. The standard account presumes that $p_{GAIN} = 0.5$ and $p_{LOSS} = 0.5$, but this assumption is not necessary and is not made here.

These decision problems are resolved by a Kahneman-Tversky investor as follows:

after a gain x_0, hold is preferred to sell if and only if $w(p_{GAIN})v_+(x_0+ h) + w(1-p_{GAIN})v_+(x_0- h) > v_+ (x_0)$

and

after a loss $-x_0$, hold is preferred to sell if and only if $w(p_{LOSS})v_-(-x_0+ h) + w(1-p_{LOSS})v_-(-x_0- h) > v_-(-x_0)$,

where v_+ and v_- are the components of the Kahneman-Tversky valuation function for gains and losses, respectively, and w is the probability weighting function.[2] The probability weighting function is a reverse-S-Shaped function of the probability, and overweights small probabilities and underweights moderate and high probabilities. The probability weighting function is presented in Figure 2. Note that these formulations of the decision problems respect the observation that "in theory, every mental account holding a stock has a reference point determined by the purchase price" (Shefrin [2005], p. 426).

Consider a Kahneman-Tversky investor with a gain x_0. As noted, the Kahneman-Tversky investor with a gain x_0 will choose hold over sell if and only if

$$w(p_{GAIN})v_+(x_0 + h) + w(1 - p_{GAIN})v_+(x_0 - h) > v_+(x_0).$$

Since the probability weighting function, $w(p_{GAIN})$, is not a simple function of p_{GAIN}, we cannot factor the forego-

TABLE 2
Payoffs after a loss $-x_0$

	Sell	HOLD	Probability
Incremental gain (h)	$-x_0$	$-x_0+ h$	p_{LOSS}
Incremental loss ($-h$)	$-x_0$	$-x_0- h$	$1 - p_{LOSS}$

ing equation to solve for p_{GAIN}. We can, however, rearrange the foregoing equation to solve for $\frac{w(p_{GAIN})}{w(1-p_{GAIN})}$ and thereby determine the critical risk value of p_{GAIN}. Since the probability weighting function is sub-additive, that is, $w(p_{GAIN})+w(1-p_{GAIN}) < 1$ for all p strictly between 0 and 1, we replace $v+(x_0)$ with $[w(p_{GAIN})+w(1-p_{GAIN})+1-w(p_{GAIN})-w(1-p_{GAIN})]\ v_+(x_0)$. The forgoing inequality then rearranges to[3]:

$$\frac{w(p_{GAIN})}{w(1-p_{GAIN})} > \frac{-[v_+(x_0-h)-v_+(x_0)]}{[v_+(x_0+h)-v_+(x_0)]} + \frac{[1-w(p_{GAIN})-w(1-p_{GAIN})]}{w(1-p_{GAIN})} \times \frac{v_+(x_0)}{[v_+(x_0+h)-v_+(x_0)]}.$$

The equality

$$\frac{w(p_{GAIN})}{w(1-p_{GAIN})} = \frac{-[v_+(x_0-h)-v_+(x_0)]}{[v_+(x_0+h)-v_+(x_0)]} + \frac{[1-w(p_{GAIN})-w(1-p_{GAIN})]}{w(1-p_{GAIN})} \times \frac{v_+(x_0)}{[v_+(x_0+h)-v_+(x_0)]}$$

determines the critical risk value of $^{p_{GAIN}}_{KT}$, denoted $^{p_{GAIN}}_{KT}$, and we will refer to

$$\frac{-[v_+(x_0-h)-v_+(x_0)]}{[v_+(x_0+h)-v_+(x_0)]} + \frac{[1-w(p_{GAIN})-w(1-p_{GAIN})]}{w(1-p_{GAIN})} \times \frac{v_+(x_0)}{[v_+(x_0+h)-v_+(x_0)]}$$

as the Kahneman-Tversky threshold for a gain x_0.

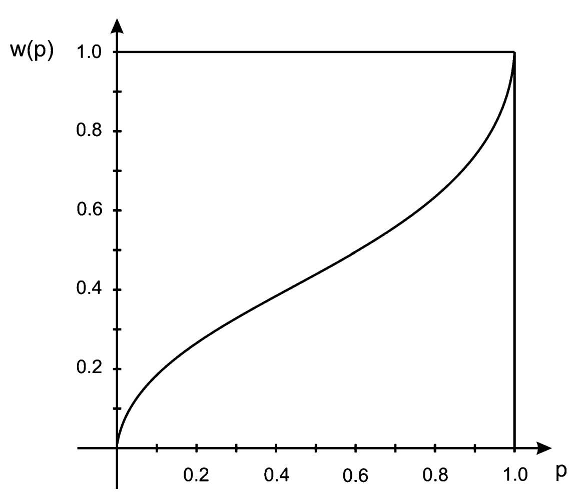

FIGURE 2 Probability weighting function.

Consider the components of the Kahneman-Tversky threshold for a gain. Since v_+ is a concave function, the ratio $\frac{-[v_+(x_0-h)-v_+(x_0)]}{[v_+(x_0+h)-v_+(x_0)]}$ is greater than unity. The ratio $\frac{[1-w(p_{GAIN})-w(1-p_{GAIN})]}{w(1-p_{GAIN})}$ is a monotonically increasing function of p_{GAIN} for any reverse-S-shaped probability weighting function $w(p_{GAIN})$. Again, since v_+ is concave, and since $v_+(x_0) > 0$, the ratio $\frac{v_+(x_0)}{[v_+(x_0+h)-v_+(x_0)]}$ is greater than zero. Therefore, the Kahneman-Tversky threshold for a gain is a monotonically increasing function of p_{GAIN}. Furthermore, the Kahneman-Tversky threshold for a gain is greater than unity for all values of p_{GAIN} since $\frac{-[v_+(x_0-h)-v_+(x_0)]}{[v_+(x_0+h)-v_+(x_0)]}$ is greater than unity and both $\frac{[1-w(p_{GAIN})-w(1-p_{GAIN})]}{w(1-p_{GAIN})}$ and $\frac{v_+(x_0)}{[v_+(x_0+h)-v_+(x_0)]}$ are greater than zero. The graph of the Kahneman-Tversky threshold for a gain x_0 is presented in Figure 3.

Similarly, under the Kahneman-Tversky decision rule, an investor with a loss will choose hold over sell if and only if

$$w(p_{LOSS})v_-(-x_0+h) + w(1-p_{LOSS})v_-(-x_0-h) > v_-(-x_0).$$

Again, since the probability weighting function is sub-additive, we replace $v_-(-x_0)$ with $[w(p_{LOSS})+w(1\text{-}p_{LOSS})+1-w(p_{LOSS})-w(1-p_{LOSS})]v_-(-x_0)$. The forgoing inequality then rearranges to:[4]

$$\frac{w(p_{LOSS})}{w(1-p_{LOSS})} > \frac{-[v_-(-x_0-h)-v_-(-x_0)]}{[v_-(-x_0+h)-v_-(-x_0)]} + \frac{[1-w(p_{LOSS})-w(1-p_{LOSS})]}{w(1-p_{LOSS})} \times \frac{v_-(-x_0)}{[v_-(-x_0+h)-v_-(-x_0)]}$$

The equality

$$\frac{w(p_{LOSS})}{w(1-p_{LOSS})} = \frac{-[v_-(-x_0-h)-v_-(-x_0)]}{[v_-(-x_0+h)-v_-(-x_0)]} + \frac{[1-w(p_{LOSS})-w(1-p_{LOSS})]}{w(1-p_{LOSS})} \times \frac{v_-(-x_0)}{[v_-(-x_0+h)-v_-(-x_0)]}$$

determines the critical risk value of p_{LOSS}, denoted p^{LOSS}_{KT}, and we will refer to

$$\frac{-[v_-(-x_0-h)-v_-(-x_0)]}{[v_-(-x_0+h)-v_-(-x_0)]} + \frac{[1-w(p_{LOSS})-w(1-p_{LOSS})]}{w(1-p_{LOSS})} \times \frac{v_-(-x_0)}{[v_-(-x_0+h)-v_-(-x_0)]}$$

as the Kahneman-Tversky threshold for a loss $-x_0$.

Consider the components of the Kahneman-Tversky threshold for a loss. Since v_- is a convex function,

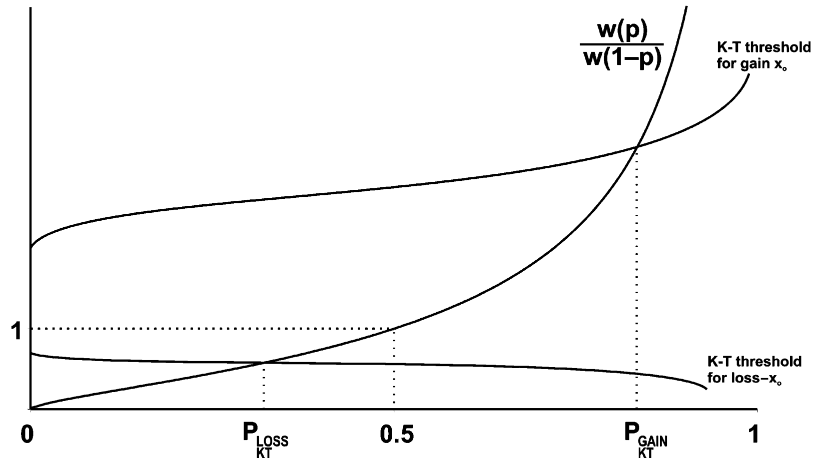

FIGURE 3 Determination of the critical risk values for the Kahneman-Tversky investor.

the ratio $\frac{-[v_-(-x_0-h)-v_-(-x_0)]}{[v_-(-x_0+h)-v_-(-x_0)]}$ is less than unity. The ratio $\frac{[1-w(p_{LOSS})-w(1-p_{LOSS})]}{w(1-p_{LOSS})}$ is a monotonically increasing function of p_{LOSS}. Since v_- is convex and $v_-(-x_0)$ is less than zero, the ratio $\frac{v_-(-x_0)}{[v_-(-x_0+h)-v_-(-x_0)]}$ is less than 0. Therefore, the Kahneman-Tversky threshold for a loss is a monotonically decreasing function of p_{LOSS}. Furthermore, the Kahneman-Tversky threshold for a loss is less than unity for all values of p_{LOSS} since $\frac{-[v_-(-x_0-h)-v_-(-x_0)]}{[v_-(-x_0+h)-v_-(-x_0)]}$ is less than unity and $\frac{v_-(-x_0)}{[v_-(-x_0+h)-v_-(-x_0)]}$ is less than zero. The graph of the Kahneman-Tversky threshold for a loss $-x_0$ is presented in Figure 3. The figure presents the critical risk values $p_{\underset{KT}{GAIN}}$ and $p_{\underset{KT}{LOSS}}$ as determined by the intersections of the Kahneman-Tversky thresholds with the function $\frac{w(p)}{w(1-p)}$.

Clearly, if $p_{\underset{KT}{LOSS}} < p_{LOSS}$ and $p_{GAIN} < p_{\underset{KT}{GAIN}}$, then the Kahneman-Tversky investor with the gain x_0 sells, and the investor with the loss $-x_0$ holds. Furthermore, it is easy to show that $p_{\underset{KT}{LOSS}} < 0.5 < p_{\underset{KT}{GAIN}}$. The two primary sub-intervals of the interval ($p_{\underset{KT}{LOSS}}$, $p_{\underset{KT}{GAIN}}$) are interesting. Note that for values of the probability p_{GAIN} where $0.5 < p_{GAIN} < p_{\underset{KT}{GAIN}}$, the investor with the gain x_0 faces an incremental gain in the amount h with a probability greater than .5 and nonetheless sells; and, for values of the probability p_{LOSS} where $p_{\underset{KT}{LOSS}} < p_{LOSS} < 0.5$, the investor with the loss $-x_0$ faces an incremental loss in the amount $-h$ with a probability greater than .5 and nonetheless holds.

Note, also, that investor behavior in these sub-intervals can be consistent with common sense. Specifically, if $p_{\underset{KT}{LOSS}} < p_{GAIN} < 0.5 < p_{\underset{KT}{GAIN}}$, then the investor with the gain x_0 faces an incremental loss in the amount $-h$ with a probability greater than .5 and sells, as common sense would suggest. Similarly, if $p_{\underset{KT}{LOSS}} < 0.5 < p_{LOSS} < p_{\underset{KT}{GAIN}}$, then the investor with the loss $-x_0$ faces an incremental gain in the amount h with a probability greater than .5 and holds, also as common sense suggests.

TIME-INDEPENDENT ACCOUNT OF THE DISPOSITION EFFECT

The foregoing analysis can be extended to a time-independent account of the disposition effect. The logic behind the disposition effect is the logic of counterfactual propositions (Dacey [1975]; Lewis [1973]). Specifically, the disposition effect claims that investors with gains sell stocks they should hold, and investors with losses hold stocks they should sell. So stated, there is no reference to time. However, the disposition effect is typically presumed to claim that investors with gains sell stocks they should have held and investors with losses hold stocks they should have sold. So stated, there is a definite reference to time, albeit without any specification of the time period. As noted at the outset, there are numerous time-dependent characterizations of the disposition effect. It remains to provide a precise account of the time-independent characterization of the disposition effect.

The foregoing analysis allows a precise account of a time-independent account of the disposition effect. The account is based on a simple idea—we can employ a von Neumann-Morgenstern decision maker as the ideal, and then we can interpret "should have done" as "what the ideal decision maker would have done." This kind of comparison between Kahneman-Tversky and von Neumann-Morgenstern decision makers is not new. It is stated in Shefrin ([2005], 421) as follows:

> The disposition effect concerns a predisposition. Given an investor's tax situation and beliefs, the disposition effect predisposes the investor to sell his or her winners more quickly but hold on to his or her losers, relative to an expected utility investor in the same situation.

Shefrin and Statman ([1985], p. 779) make a similar statement: "Prospect theory suggests the hypothesis that investors display a disposition to sell winners and ride losers when standard theory suggests otherwise." Note that the 'standard theory' referred to in Shefrin and Statman [1985] is the tax-based theory advanced by Constantinides [1983, 1984].

Since the valuation function of prospect theory is a piece-wise defined von Neumann-Morgenstern utility function (Fishburn and Kochenberger [1979]), we will retain the valuation function of the foregoing analysis of the Kahneman-Tversky investor for the analysis of the von Neumann-Morgenstern investor. First, this assumption is consistent with the view, taken here, that it is a matter of fact that the majority of decision makers have S-shaped valuation/utility functions. Second, this assumption will allow us to examine the role of the probability weighting function in greater detail in that while both the Kahneman-Tversky and von Neumann-Morgenstern investors employ the same valuation function, only the Kahneman-Tversky investor employs the probability weighting function. Put differently, the von Neumann-Morgenstern investor is the Kahneman-Tversky investor with $w(p) = p$. Thus, differences in behavior between the Kahneman-Tversky and von Neumann-Morgenstern investors are due to the probability weighting function.

Again, the decision problems after a gain and after a loss are given in Table 1 and Table 2, respectively. These decision problems are resolved by a von Neumann-Morgenstern investor as follows:

after a gain x_0, hold is preferred to sell if and only if
$p_{GAIN}v_+(x_0 + h) + (1 - p_{GAIN})v_+(x_0 - h) > v_+(x_0)$;

and

after a loss $-x_0$, hold is preferred to sell if and only if
$p_{LOSS}v_-(-x_0 + h) + (1 - p_{LOSS})v_-(-x_0 - h) > v_-(-x_0)$

The analysis of these decision problems for the von Neumann-Morgenstern investor follows the pattern of the analyses for the Kahneman-Tversky investor. While $p_{GAIN}v_+(x_0+h) + (1 - p_{GAIN})v_+(x_0- h) > v_+(x_0)$ can be solved for p_{GAIN}, we replace $v_+(x_0)$ with $[p_{GAIN}+ (1-p_{GAIN})]v_+(x_0)$ and solve for $\frac{p_{GAIN}}{1-p_{GAIN}}$. The forgoing inequality then rearranges to:

$$\frac{p_{GAIN}}{(1 - p_{GAIN})} > \frac{-[v_+(x_0 - h) - v_+(x_0)]}{[v_+(x_0 + h) - v_+(x_0)]}$$

The equality $\frac{p_{GAIN}}{(1-p_{GAIN})} = \frac{-[v_+(x_0-h)-v_+(x_0)]}{[v_+(x_0+h)-v_+(x_0)]}$ determines the critical risk value of p_{GAIN}, denoted $p_{\substack{GAIN\\vNM}}$, under the von Neumann-Morgenstern decision rule, and we will refer to $\frac{-[v_+(x_0-h)-v_+(x_0)]}{[v_+(x_0+h)-v_+(x_0)]}$ as the von Neumann-Morgenstern threshold for a gain x_0. Note that the von Neumann-Morgenstern threshold for a gain is the first component of the Kahneman-Tversky threshold for a gain. Since v_+ is concave, the von Neumann-Morgenstern threshold for a gain is greater than unity and is constant for all values of p_{GAIN}.

A von Neumann-Morgenstern investor with a loss $-x_0$ will choose hold over sell if and only if

$$p_{LOSS}v_-(-x_0 + h) + (1 - p_{LOSS})v_-(-x_0 - h) > v_-(-x_0)$$

We replace $v_-(x_0)$ with $[p_{LOSS}+ (1 - p_{LOSS})]v_-(x_0)$. The forgoing inequality then rearranges to[6]:

$$\frac{p_{LOSS}}{(1 - p_{LOSS})} > \frac{-[v_-(-x_0 - h) - v_-(-x_0)]}{[v_-(-x_0 + h) - v_-(-x_0)]}$$

The equality $\frac{p_{LOSS}}{(1-p_{LOSS})} = \frac{-[v_-(-x_0-h)-v_-(-x_0)]}{[v_-(-x_0+h)-v_-(-x_0)]}$ determines the critical risk values of p_{LOSS}, denoted $p_{\substack{LOSS\\vNM}}$, under the von Neumann-Morgenstern decision rule, and we will refer to $\frac{-[v_-(-x_0-h)-v_-(-x_0)]}{[v_-(-x_0+h)-v_-(-x_0)]}$ as the von Neumann-Morgenstern threshold for a loss $-x_0$. Note that the von Neumann-Morgenstern threshold for a loss is the first component of the Kahneman-Tversky threshold for a loss. Since v_- is convex, the von Neumann-Morgenstern threshold for a loss is less than unity and is constant for all values of p_{LOSS}. The intersections of the von Neumann-Morgenstern thresholds with the function $\frac{p}{1-p}$ determine the critical risk values $p_{\substack{GAIN\\vNM}}$ and $p_{\substack{LOSS\\vNM}}$, as shown in Figure 4. The figure presents the critical risk values $p_{\substack{GAIN\\vNM}}$ and

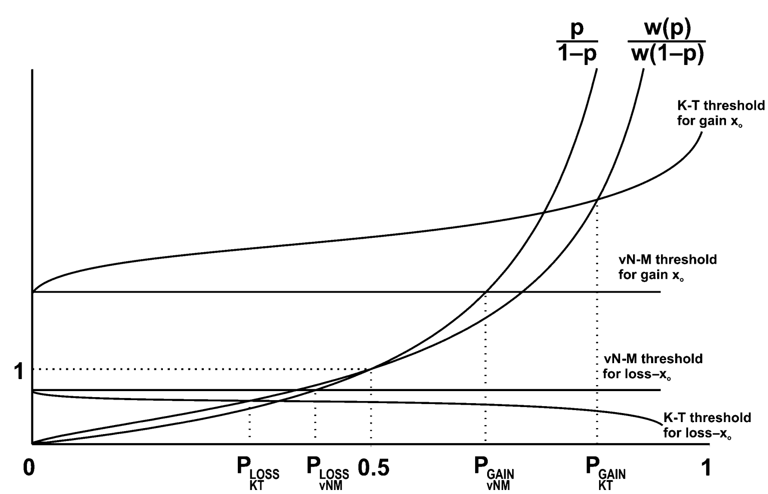

FIGURE 4 Determination of the critical risk values for the Kahneman-Tversky and von Neumann-Morgenstern investors.

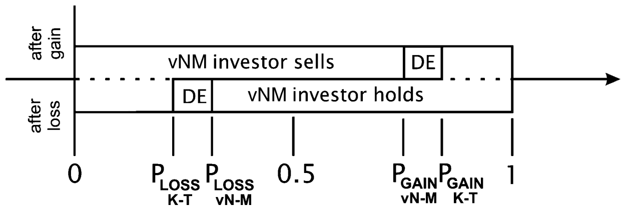

FIGURE 5 Disposition Effect as a Function of Probability. DE–disposition effect.

$p_{\underset{vNM}{LOSS}}$ as determined by the intersections of the von Neumann-Morgenstern thresholds with the function $\frac{p}{1-p}$, along with the critical risk values $p_{\underset{KT}{GAIN}}$ and $p_{\underset{KT}{LOSS}}$ previously discussed.

Clearly, if $p_{\underset{vNM}{LOSS}} < p_{LOSS}$ and $p_{GAIN} < p_{\underset{vNM}{GAIN}}$, then the von Neumann-Morgenstern investor with the gain x_0 sells, and the investor with the loss $-x_0$ holds. Furthermore, the critical risk values for the Kahneman-Tversky and von Neumann-Morgenstern investors partition the unit interval as follows:

$$0 < p_{\underset{KT}{LOSS}} < p_{\underset{vNM}{LOSS}} < 0.5 < p_{\underset{vNM}{GAIN}} < p_{\underset{KT}{GAIN}} < 1$$

The most interesting results pertain to the intervals $(p_{\underset{vNM}{GAIN}}, p_{\underset{KT}{GAIN}})$ and $(p_{\underset{KT}{LOSS}}, p_{\underset{vNM}{LOSS}})$. Consider values of p_{GAIN} in the interval $(p_{\underset{vNM}{GAIN}}, p_{\underset{KT}{GAIN}})$. Here the Kahneman-Tversky investor with the gain x_0 sells, while the von Neumann-Morgenstern investor with the gain x_0 holds. Thus, we have the interval on p_{GAIN} where the Kahneman-Tversky investor exhibits the disposition effect given a gain x_0, vis-à-vis the von Neumann-Morgenstern investor. Contrariwise, consider values of p_{LOSS} in the interval $(p_{\underset{KT}{LOSS}}, p_{\underset{vNM}{LOSS}})$. Here the Kahneman-Tversky investor with the loss $-x_0$ holds, while the von Neumann-Morgenstern investor with the loss $-x_0$ sells. Thus, we have the interval on p_{LOSS}, where the Kahneman-Tversky investor exhibits the disposition effect given a loss $-x_0$, vis-à-vis the von Neumann-Morgenstern investor.

We will say that the Kahneman-Tversky investor exhibits the disposition effect if and only if $p_{\underset{vNM}{GAIN}} < p_{GAIN} < p_{\underset{KT}{GAIN}}$ or $p_{\underset{KT}{LOSS}} < p_{LOSS} < p_{\underset{vNM}{LOSS}}$. This amounts to stating that the Kahneman-Tversky investor holds when he or she should sell and sells when he or she should hold, where 'should' is characterized by the behavior of the von Neumann-Morgenstern investor. So restated, the disposition effect can be seen in standard counterfactual terms, without recourse to time. These results are presented schematically in Figure 5.

The foregoing analysis follows the suggestions of Shefrin [2005] and Shefrin and Statmen [1985] to draw a comparison between investor behavior based on prospect theory and expected utility theory. We obtain an interesting result if we go one step further and draw a distinction between expected utility theory and expected value theory. This additional step allows us to examine the effects of the valuation function in the absence of the probability weighting function since $w(p) = p$ for both the von Neumann-Morgenstern investor and the Pascal investor. A loss averse Pascal investor has the valuation function $v_+(x) = x$ for $x \geq 0$ and $v_-(x) = -\alpha(-x)$ for $x \leq 0$, where $\alpha > 1$. Following the logic employed earlier, we find that the thresholds for the Pascal investor are transformations of the von Neumann-Morgenstern thresholds, as follows:

$$\frac{-[v_+(x_0 - h) - v_+(x_0)]}{[v_+(x_0 + h) - v_+(x_0)]} \text{ becomes}$$

$$\frac{-[(x_0 - h) - (x_0)]}{[(x_0 + h) - (x_0)]} = \frac{h}{h} = 1,$$

and

$$\frac{-[v_-(-x_0 - h) - v_-(-x_0)]}{[v_-(-x_0 + h) - v_-(-x_0)]} \text{ becomes}$$

$$\frac{-[-\alpha(x_0 + h) - (-\alpha(x_0))]}{[-\alpha(x_0 - h) - (-\alpha(x_0)]} = \frac{\alpha h}{\alpha h} = 1$$

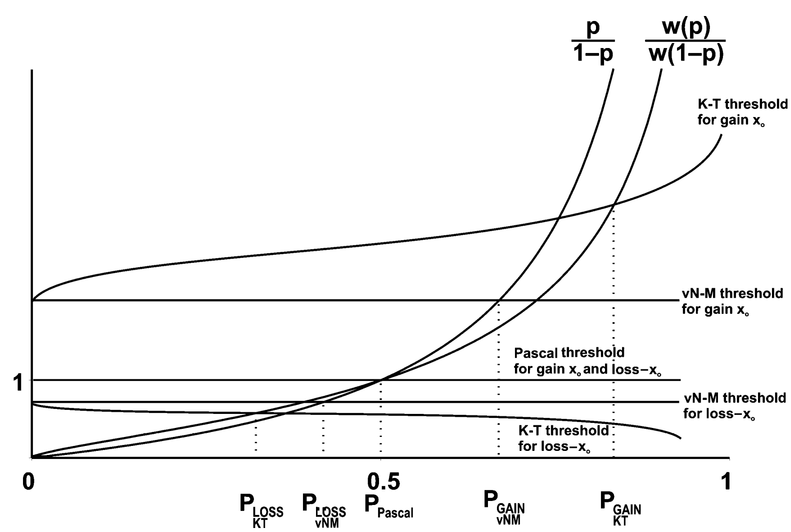

FIGURE 6 Determination of the critical risk values for the Kahneman-Tversky, von Neumann-Morgenstern, and Pascal investors.

Thus, the Pascal thresholds intersect the function $\frac{p}{1-p}$ at $\frac{p}{1-p} = 1$, and thereby at $p = 0.5$. Therefore, the critical risk values for the Pascal investor are $p_{\underset{\text{Pascal}}{\text{GAIN}}} = p_{\underset{\text{Pascal}}{\text{LOSS}}} = 0.5$, and the Pascal investor holds after a gain x_0 if and only if $p_{\text{GAIN}} > 0.5$, and the Pascal investor sells after a loss $-x_0$ if and only if $p_{\text{LOSS}} < 0.5$. This result, together with the analogous results for the Kahneman-Tversky investor and the von Neumann-Morgenstern investor are shown in Figure 6.

We can now distinguish the roles of the valuation and probability weighting functions in explaining the disposition effect. If $0.5 < p_{\text{GAIN}} < p_{\underset{\text{vNM}}{\text{GAIN}}} < p_{\underset{\text{KT}}{\text{GAIN}}}$ and $p_{\underset{\text{KT}}{\text{LOSS}}} < p_{\underset{\text{vNM}}{\text{LOSS}}}$ $p_{\text{LOSS}} < 0.5$, then both the Kahneman-Tversky investor and the von Neumann-Morgenstern investor exhibit the disposition effect. However, if $0.5 < p_{\underset{\text{vNM}}{\text{GAIN}}} < p_{\text{GAIN}} < p_{\underset{\text{KT}}{\text{GAIN}}}$ and $p_{\underset{\text{KT}}{\text{LOSS}}} < p_{\text{LOSS}} < p_{\underset{\text{vNM}}{\text{LOSS}}} < 0.5$, then only the Kahneman-Tversky investor exhibits the disposition effect. Thus, we have the two-part result that if

$$0.5 < p_{\text{GAIN}} < p_{\underset{\text{vNM}}{\text{GAIN}}} < p_{\underset{\text{KT}}{\text{GAIN}}} \quad \text{and} \quad p_{\underset{\text{KT}}{\text{LOSS}}} < p_{\underset{\text{vNM}}{\text{LOSS}}} < p_{\text{LOSS}} < 0.5,$$

then the disposition effect can be explained by the S-shaped valuation function alone, whereas if

$$0.5 < p_{\underset{\text{vNM}}{\text{GAIN}}} < p_{\text{GAIN}} < p_{\underset{\text{KT}}{\text{GAIN}}} \quad \text{and} \quad p_{\underset{\text{KT}}{\text{LOSS}}} < p_{\text{LOSS}} < p_{\underset{\text{vNM}}{\text{LOSS}}} < 0.5,$$

then the disposition effect must be explained by the S-shaped valuation function together with the probability weighting function.

CONCLUSION

We provided a detailed explanation of the disposition effect based on a straightforward application of prospect theory. The analysis employed a comparison of Kahneman-Tversky and von Neumann-Morgenstern investors to give precise, and time-independent, definitions of "sell too soon" and "hold too long." We also specified when the explanation of the disposition effect requires only the S-shaped valuation function of prospect theory and when the explanation requires both the valuation function and the probability weighting function.

Our analysis considered only the standard case where the incremental gain or loss is less than the initial gain or loss. We can call this the low volatility case. It remains to extend this analysis to the case where the incremental gain or loss is greater than the initial gain or loss. This, of course, would be called the high volatility case.

Oehler et al. [2003] maintain that the main reason the market does not collapse or become illiquid in the face of disposition investors' reluctance to trade is the coexistence of momentum and contrarian traders. Thus, it remains to extend the foregoing analysis to accommodate precise definitions of momentum and contrarian investors and to examine the claim made by Oehler et al. [2003].

NOTES

1. The Kahneman-Tversky valuation function is:

$$v(x) = \begin{cases} x^{\alpha} & x \geq 0 \\ -\lambda\,(-x)^{\beta} & x \leq 0 \end{cases}$$

with $0 < \alpha, \beta < 1$ and $\lambda > 1$. The parameter estimates are $\alpha = \beta = .88$ and $\lambda = 2.25$ (Tversky and Kahneman [1992], p. 311).

2. The Kahneman-Tversky probability weighting function is

$$w(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{\frac{1}{\gamma}}}$$

with $0 < \gamma < 1$. The parameter estimates are $\gamma = 0.61$ for gains and $\gamma = .69$ for losses (Tversky and Kahneman [1992], p. 312). Alternative formulations of the probability weighting function are advanced by Prelec [1998] and Gonzalez and Wu [1999].

It is important to note that the probability weighting function specified in Tversky and Kahneman [1992] is defined over cumulative probabilities so as to make cumulative prospect theory consistent with stochastic dominance. However, if we assume that the w-function for gains is identical to the w-function for losses, as was assumed in the original version of prospect theory, then "the present (cumulative prospect) theory coincides with the original version for all two-outcome prospects . . . " (Tversky and Kahneman [1992], p. 302). The analysis presented here presumes that the w-function for gains is identical to the w-function for losses. Therefore, we can extract the relevant comparisons between Kahneman-Tversky investors and two variations on the Kahneman-Tversky investor. Since the disposition problem involves only two-outcome prospects, the account of prospect theory employed here melds the original [1979] and the cumulative [1992] versions of the theory.

3. The derivation is as follows:
$w(p_{\text{GAIN}})v_+(x_0+h) + w(1-p_{\text{GAIN}})v_+(x_0-h) > v_+(x_0)$ by definition;
$w(p_{\text{GAIN}})\,v_+(x_0+h) + w(1-p_{\text{GAIN}})v_+(x_0-h) >$
$[w(p_{\text{GAIN}}) + w(1\text{-}p_{\text{GAIN}}) + 1 - w(p_{\text{GAIN}}) - w(1\text{-}p_{\text{GAIN}})]v_+(x_0)$ by substitution;
$w(p_{\text{GAIN}})[v_+(x_0+h) - v_+(x_0)] + w(1\text{-}p_{\text{GAIN}})[v_+(x_0-h) - v_+(x_0)] >$
$[1 - w(p_{\text{GAIN}}) - w(1-p_{\text{GAIN}})]\,v_+(x_0)$ by subtraction;
$w(p_{\text{GAIN}})[v_+(x_0+h) - v_+(x_0)] >$
$-w(1-p_{\text{GAIN}})[\,v_+(x_0-h) - v_+(x_0)] + [1 - w(p_{\text{GAIN}}) - w(1-p_{\text{GAIN}})]v_+(x_0)$ by subtraction;

$$\frac{w(p_{\text{GAIN}})}{w(1-p_{\text{GAIN}})} > \frac{-[v_+(x_0-h) - v_+(x_0)]}{[v_+(x_0+h) - v_+(x_0)]}$$

$$+ \frac{[1 - w(p_{\text{GAIN}}) - w(1-p_{\text{GAIN}})]}{w(1-p_{\text{GAIN}})} \frac{v_+(x_0)}{[v_+(x_0+h) - v_+(x_0)]}$$

by division.

4. The derivation is as follows:
$p_{\text{GAIN}}v_+(x_0+h) + (1-p_{\text{GAIN}})v_+(x_0-h) > v_+(x_0)$ by definition;
$p_{\text{GAIN}}v_+(x_0+h) + (1-p_{\text{GAIN}})v_+(x_0-h) > [p_{\text{GAIN}} + (1-p_{\text{GAIN}})]v_+(x_0)$ bysubstitution;
$p_{\text{GAIN}}v_+(x_0+h) - v_+(x_0)] + (1-p_{\text{GAIN}})[v_+(x_0-h) - v_+(x_0)] > 0$ by subtraction;
$p_{\text{GAIN}}[v_+(x_0+h) - v_+(x_0)] > -(1-p_{\text{GAIN}})[v_+(x_0\text{-}h) - v_+(x_0)]$ by subtraction;

$$\frac{p_{\text{GAIN}}}{(1-p_{\text{GAIN}})} > \frac{-[v_+(x_0-h) - v_+(x_0)]}{[v_+(x_0+h) - v_+(x_0)]}$$

by division.

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